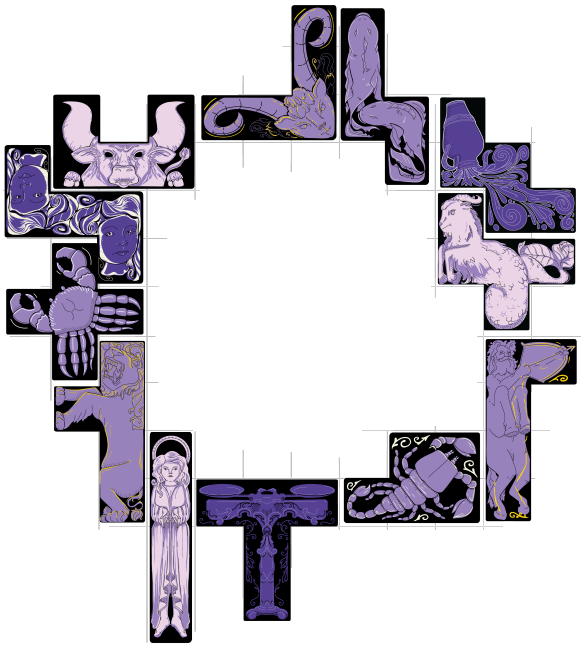
Fence Challenge

By The Fence Challenge Team
www.erikaroldan.net/fencechallenge

Have you ever heard about the Fence Challenge? You need the 12 *pentominoes*, each one linked to a zodiac sign. Your mission: using some of the 12 zodiac signs or all of them to build a gate on the square grid trying to enclose as much area as possible.



This is a **Citizen Science** project: your answer may help us go a step further in solving these open mathematical problems. We will store the highest records with solutions coming in worldwide. So, are you ready to dive in and give it a shot? Let's see what you've got!

Participants:

The Fence Challenge is suitable for participants aged 6 and up. However, younger participants could engage and understand the activity with the appropriate help and supervision. No prior math knowledge is required to play. Using the mixed reality feature that we have developed for this activity, participants do not even need to know how to count! The game can be played solo or collaboratively in teams of two or more.

Preparations:

We have developed three game modes for you:

- Hands-on (no technology required),
- Video game (no printing required), and
- Mixed reality (technology and printing required, but no counting required!)

For the mixed reality mode, you will need:

- Printed templates (tiles and board game). You can find the templates at the end of this document.
- Scissors.
- Transparent tape.
- A smartphone, tablet, or laptop, with internet access, and a camera.
- *(optional)* Pencil colors, markers, crayons, etc. to personalize your pentominoes.

If you choose to play hands-on mode, an electronic device is not required. For the video game mode, you only need a smartphone, tablet, or laptop, with internet access.

Activity

If you are choosing the mixed reality mode, follow the activity instructions as written.

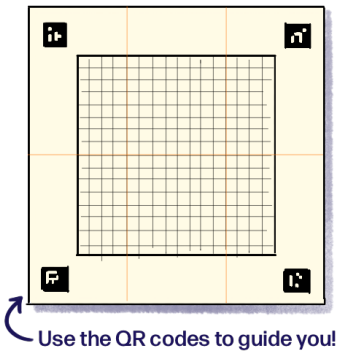
If you are choosing the hands-on mode, follow parts *1. Set up* and *2. Time to play*. Once you have your fence, practice those counting skills to find out how much area you have enclosed.

Finally, if you want to play the video game, go to part *3. Using the Fence Challenge app*, to find the link to the Fence Challenge app.

1. Set up

1. Print the pentominoes and the board. It happens that there are a total of 12 different pentominoes. As many as zodiac signs! So we decided to pair each one of the pentominoes with each zodiac sign.
2. Cut out the pentominoes. Feel free to paint and personalize your pentominoes. Just make sure to **not draw on top of the QR codes**.

3. For the board: assemble the six pieces as shown below. You can use tape to make sure the pieces stay in place.

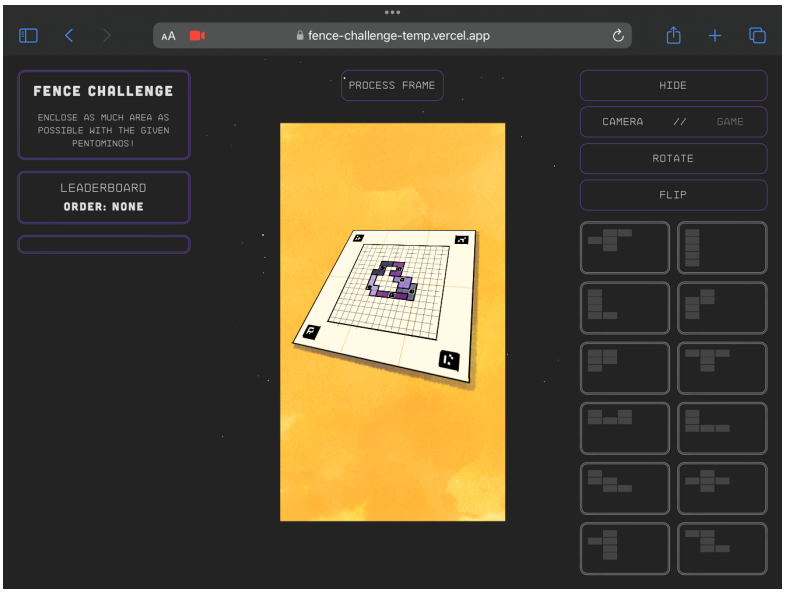


2. Time to play

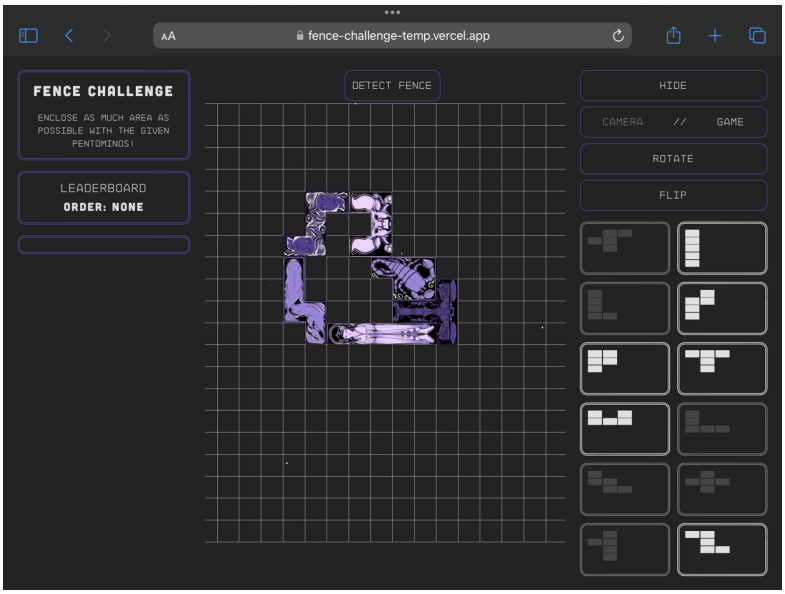
1. Choose your set of favorite pentominoes. **Caution! For this challenge, we are asking you to NOT use a zodiac sign more than once.**
2. Use them to enclose an area. Here are some examples of what is considered to be enclosed, and what is not.

3. Using the Fence Challenge app

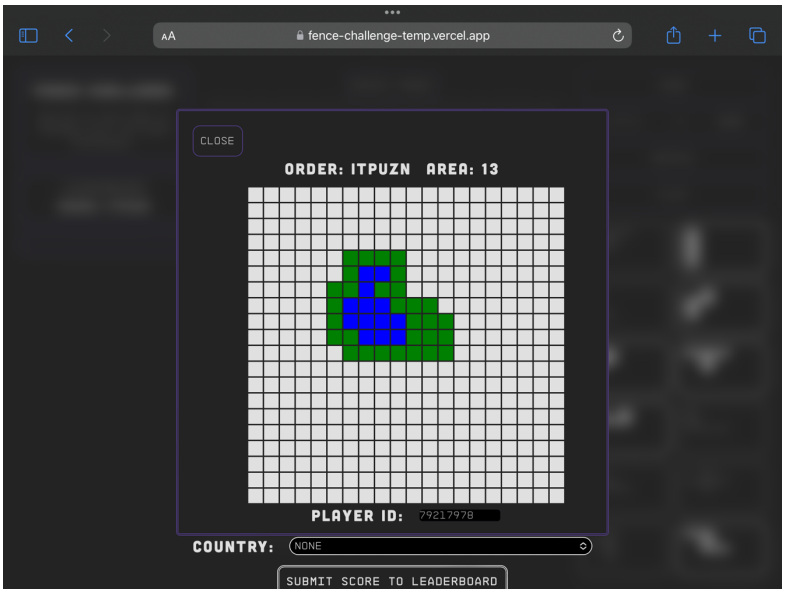
1. Open your favorite browser on your device (smartphone, tablet, etc). Go to fence-challenge-temp.vercel.app.
2. Select “Start App using camera”. Use your camera to take a picture of your fence. **Make sure the corners of the board are visible!** Once you have a nice shot of your board, click on “Process frame” to take the picture.



3. You can also explore the Game mode to build your fence. Once you are done, select “Detect fence” to see your result.



4. The app will recognize your setup and compute the area of the enclosed section.

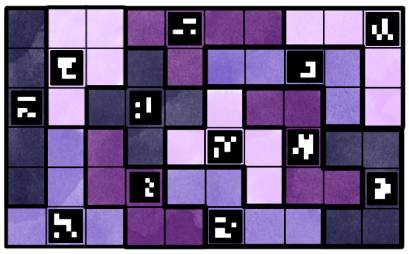


5. **We want to hear from you!** Choose your country and share your score with us. This is a very important step. As we mentioned at the beginning, this is a **Citizen Science** project. Your score helps us to get one step closer to understanding this problem.

Collaborative mode

As we mentioned, this game can be played solo or in collaborative mode. Let us propose to you one of our favorite ways to play in teams.

1. Start with an initial configuration of a set of pentominoes on the board. The edges of the pentominoes must align with the grid lines of the board. Here is our suggestion for an initial position:

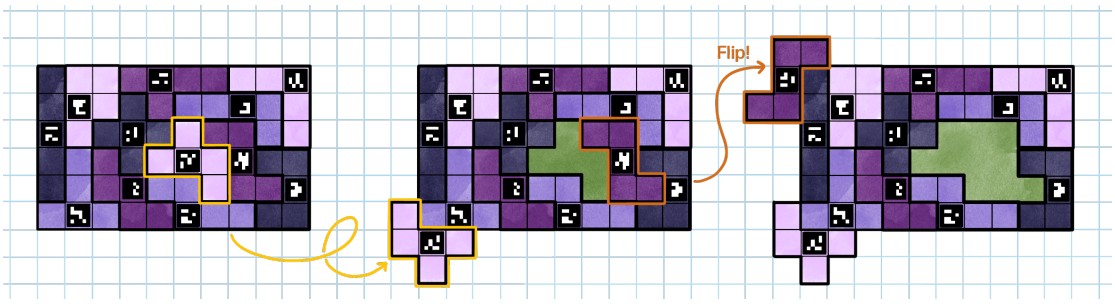


   But you can come up with your favorite initial position, using your favorite set of pentominoes.

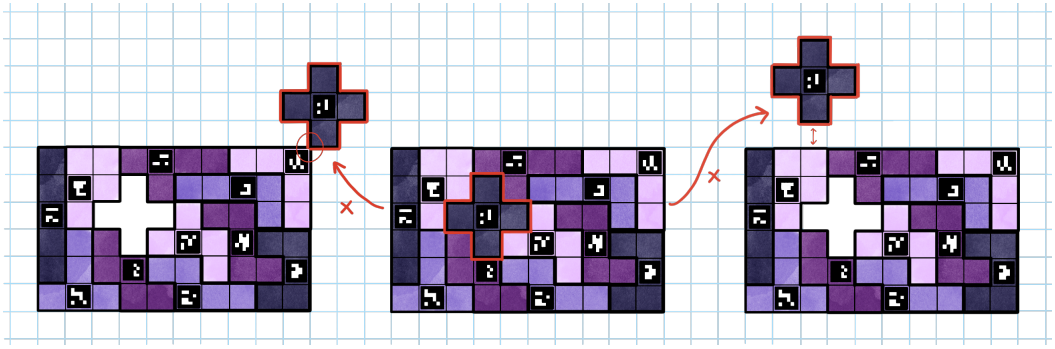
2. By turns, each player takes a pentomino, and places it somewhere else in the plane, so that the pentomino shares **at least one edge** with some other pentomino in the board, creating a fence. You can rotate the piece, move it, or flip it. If you are using our printable material, this

means you can interchange pieces that belong to the same Zodiac sign.

Here we show you an example of what the beginning of a game can look like:



Here are some examples of what movements are not allowed:



3. The game continues until everyone passes, and you have created a fence with the biggest possible enclosed area.

Create and Share!
We want to see your beautiful work! Share photos of your fence on your favorite social media using the hashtags #idm314, #idm314FenceChallenge.

Mathematical background and resources

A set of pentomino fence puzzles was first presented by Feser in 1968 [Fes68]. One of them asking to find the greatest enclosed area by the 12 pentominoes, gained popularity when it appeared in the famous Gardner's Mathematical Games column [Gar73]. In this column, Gardner presented his best attempt, of area 127 squares. He challenged the readers to find a solution to the fence challenge by matching what was then believed to be the maximal possible area, of area 128. This maximal area was found by Knuth who communicated it directly to Gardner and who also claimed to have a sketch of proof that it was indeed the maximal solution.

The first proof that 128, is indeed, the maximum possible area to be enclosed with the 12 pentominoes was given by Takakazu Shimauchi in 1978 [Shi78]. Until 2023, this result was only available in Japanese. This Citizen Science project is based on the theoretical and computational results found by Langlois-Rémillard, Müßig, and Roldan in [LRMR24]. They have also made available a translation in English of the original paper by Takakazu. In particular, they provided a database listing all configurations that can reach a maximum total area of 126, 127, and 128. They have also proved that all configurations reaching maximum area will fit in a 20×20 grid, which justifies the size that we have selected for the Fence Challenge.

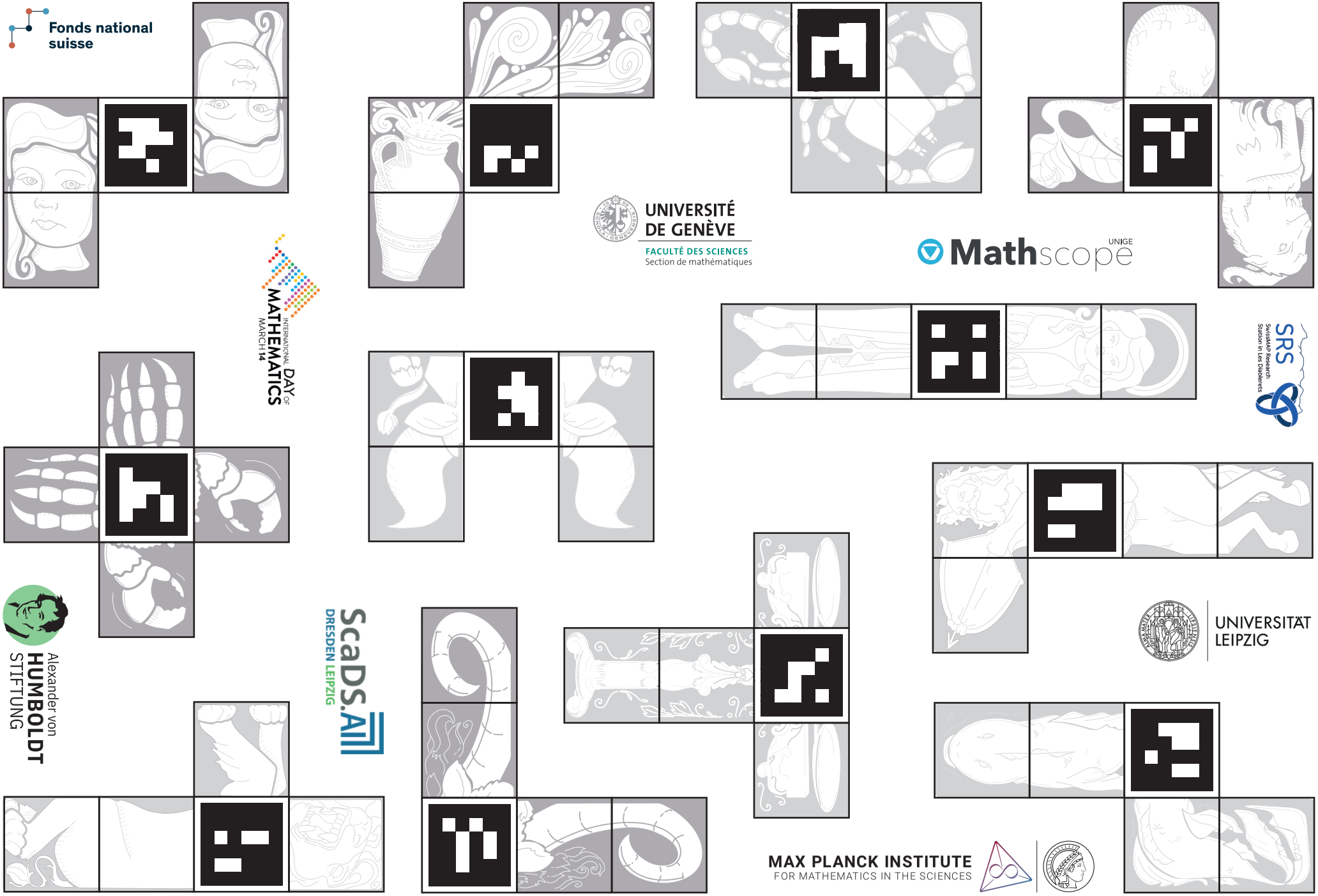
References

[Fes68] Fr V Feser. Pentomino farms. *Journal of Recreational Mathematics*, 1:675–682, 1968.

[Gar73] Martin Gardner. Mathematical games. *Scientific American*, 228(5):102—107, 1973.

[LRMR24] Alexis Langlois-Rémillard, Mia Müßig, and Érika Roldán. Extremal fence problems with polyominoes, 2024+. Work in preparation.

[Shi78] Takakazu Shimauchi. Pentomino farm. *Sugaku Seminar*, pages 11–16, March 1978.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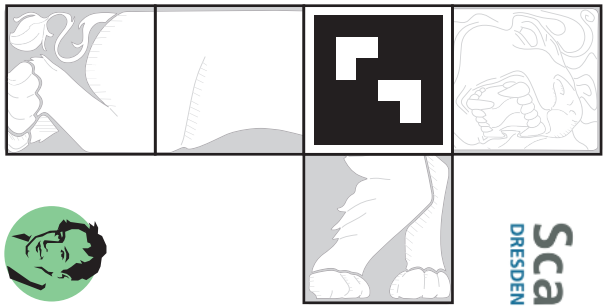

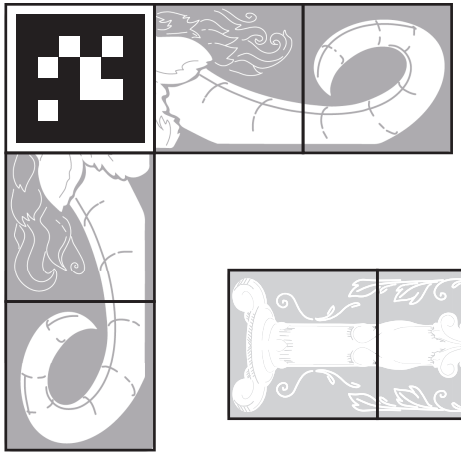

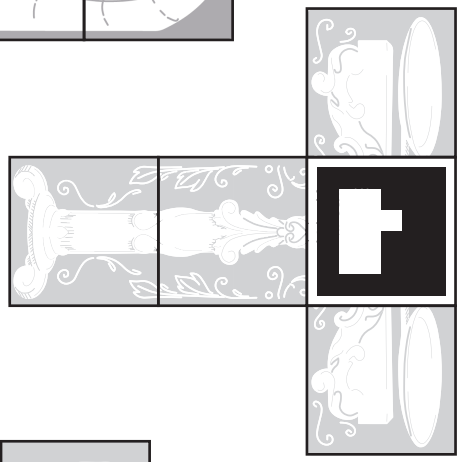

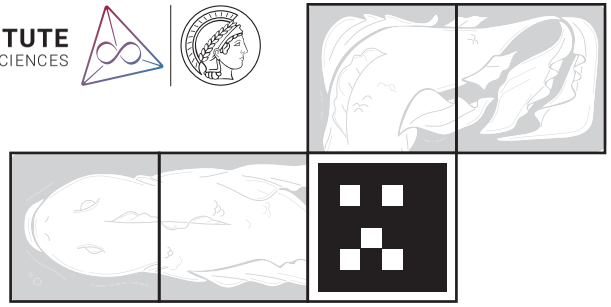

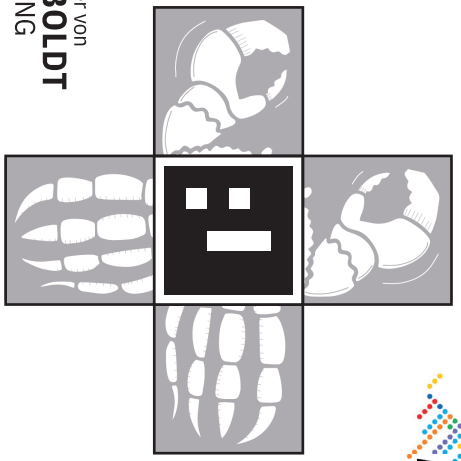

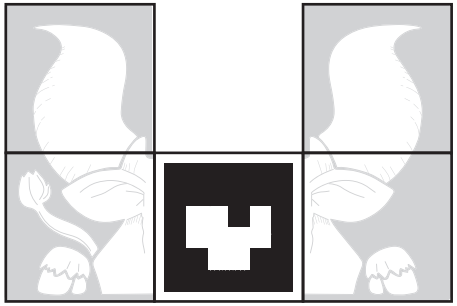

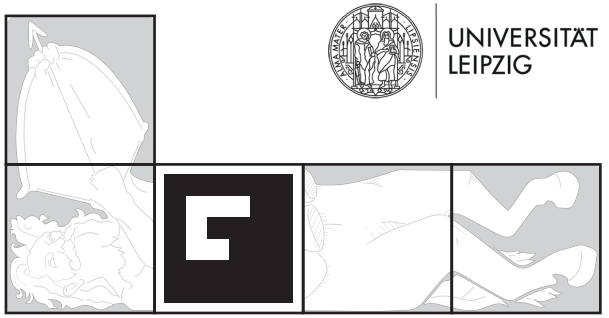

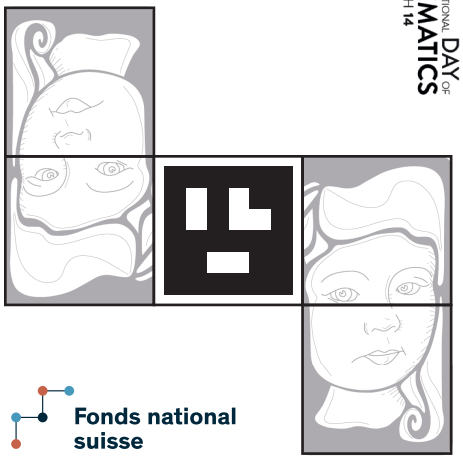

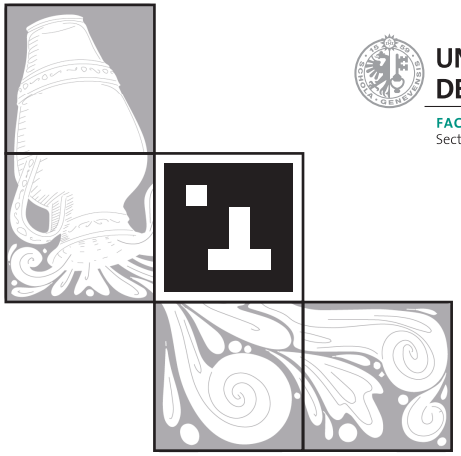

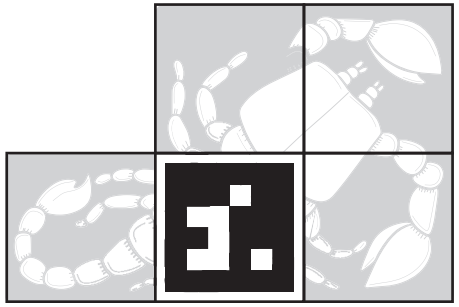

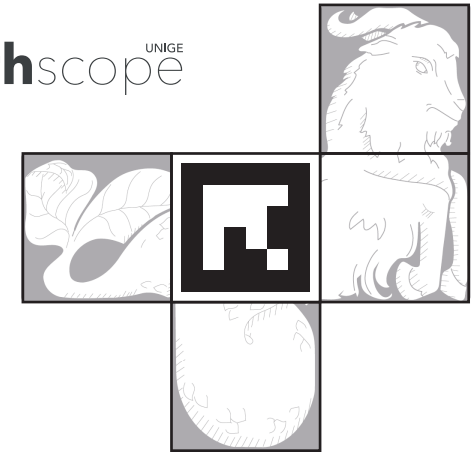

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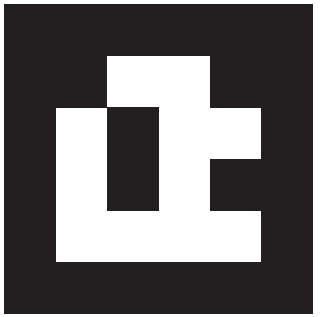

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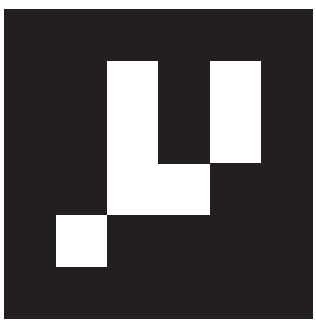

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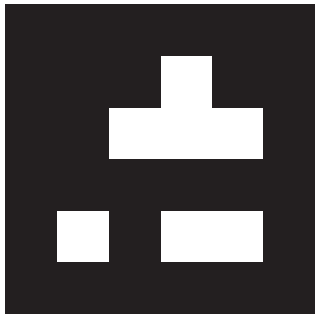

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